RAFAY HASAN ME-009



# Lecture #013

## Sensing & Actuation

## Chap 3 & 5 Liu & Chang

$$n = p = n_i$$

- $n$: electron concentration
- $p$: hole concentration
- $n_i$: intrinsic

$n_i$ for (Si) at room temp is

$$n_i = 1.5 \times 10^{10} / cm^3$$

($n_0$ and $p_0$ are different from $n_i$) For doped semiconductor piece, the concentration of carriers ($n_0$ and $p_0$) are different from $n_i$. However, it is proven that the conc. of electron and holes under thermal equilibrium are simply

$$n_0 p_0 = n_i^2$$

$n_0$ = donor = electron excess = Pho phorus

Group V

$p_0$ = acceptor = exam bles: Boron

Ohm's law

$E = \rho J$ ← bulk resistivity

$E$ → applied Electric-Field. $J$ → Current density

$\sigma = \frac{1}{\rho}$ → resistivity

$\sigma$ → Conductivity

So,

$$J = \sigma E$$

$$\sigma = J/E$$

Figure 3.1

$$\sigma = \frac{J}{E} = \frac{\frac{I_A}{A}}{E} = \frac{\frac{Q}{At}}{E} = \frac{\frac{nAdq}{At}}{E} = \frac{n\bar{V}q}{E}$$

$$\sigma = n\mu_n q$$

$\therefore q = 1.6 \times 10^{-19} C$

$$\rho = \frac{1}{\sigma} = \frac{1}{\sigma_n + \sigma_p}$$ ← Total Conductivity

($\sigma_n$ ↓ conc. electrons, $\sigma_p$ holes)

$$= \frac{1}{q(\mu_n n_i^2 + \mu_p p)}$$

$\mu_n$ speed of electron, $\mu_p$ speed of holes

## Example 3.2

$n_i = 1.5 \times 10^{10} / cm^3$

$n_0 = 10^{18} cm^{-3}$ (Phosphorus donor of electron so group V)

$\mu_n = 1350 \ cm^2/Vs$ (donor speed of electron)

$\mu_p = 480\ cm^2/V\text{-}s$ (acceptor holes speed) (2)

$\rho = ?$

Formula

$$\rho = \frac{1}{\sigma} = \frac{1}{q(\mu_n n_0 + \mu_p p_0)}$$

So, for $p_0 = \frac{n_i^2}{n_0}$

from $n_i^2 = n_0 p_0$

$$\text{So, } p_0 = \frac{(1.5 \times 10^{10})^2}{10^{18}} = 225\ cm^{-3}$$

Now,

$$\rho = \frac{1}{\sigma} = \frac{1}{1.6 \times 10^{-19}(1350 \times 10^{18} + 480 \times 225)}$$

$$\rho = 4.629 \times 10^{-3}\ \frac{V.s.cm}{C}$$

$$= 0.0046\ \frac{V \cdot cm}{A}$$

$$\rho = 0.0046\ \Omega \cdot cm$$

## Mechanical Properties

$\sigma = F/A$

Stress = $N/m^2$ or Pa (unit)

Hookes law $\sigma = E\ S$ (Poisson ratio $\nu$)

↳ (Modulus of Elasticity or Young's modulus)

## Example 3.4

Given

$F = 10mN$

$\gamma = 100 N/m^2$

Stress

$$\sigma = \frac{F}{A} = \frac{10 \times 10^{-3}}{\pi \left(\frac{100 \times 10^{-6}}{2}\right)2}$$

$\sigma = 1.27 \times 10^6 \ N/m^2$

Strain — Hooke's law

$$S = \frac{\sigma}{E} = \frac{1.37 \times 10^6}{130 \times 10^9}$$

$$S = 9.79 \times 10^{-6}$$

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For Cantilever beam

$$I = \frac{WL^3}{12} \qquad I = \frac{\pi R^4}{4}$$

Example 3.7

1st

$$I_1 = \frac{100 \times 10^{-6} \times (5 \times 10^{-6})^3}{12}$$

$$I_1 = 1.04 \times 10^{-21}$$

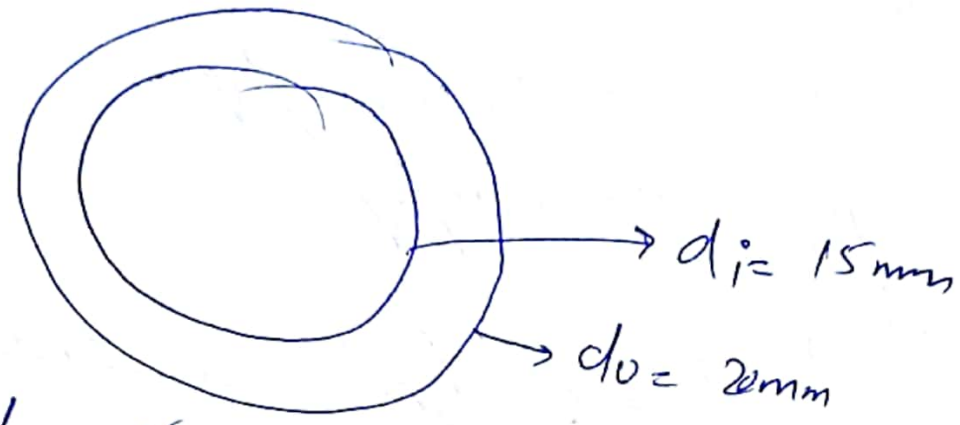
For 2nd

$$I_2 = \frac{50 \times 10^{-6} \times (8 \times 10^{-6})^3}{12}$$

$$I_2 = 2.13 \times 10^{-21}$$

$I_2 > I_1$ the second beam is stiffer judging

$E = 210\ GPa = 210 \times 10^3\ N/mm^2$




$d_i = 15mm$

$d_o = 20mm$

$$I = \frac{\pi R^4}{4}$$

$$= \frac{\pi}{4}\left(R_o^4 - R_i^4\right)$$

So, $R_o^4 = \left(\frac{20}{2}\right)^4 = 100\ mm = 10\ m$

$R_i^4 = \left(\frac{15}{2}\right)^4 = 3164.0625\ mm = 3.164\ m$

$$= \frac{\pi}{4}(10 - 3.164)$$

$I = 5.368\ Nm^2$

## Problem 3 Design

Given:-

$L = 100\ \mu m = 100 \times 10^{-6}\ m = 0.01cm$

$W = 2\mu m = 2 \times 10^{-6}\ m$

$t = 0.5\ \mu m = 0.5 \times 10^{-6}\ m$

$n_0 = 10^{17}$ atoms / $cm^3$

$n_i = 1.5 \times 10^{10}$ / $cm^3$ – for silicon

$\mu_n = 1350\ cm^2/V_s$

$\mu_p = 480\ cm^2/V_s$

Find :- $p_0 = ?$ (holes concentration?)

$n$ = electron concentration

$\rho = ?$ resistivity

$R = ?$ Total resistance

Solution

$$n_0 p_0 = n_i^2$$

$$\therefore p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{(10^{17})} = 2.25 \times 10^3\ cm^{-3}$$

So,

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)} = \frac{1}{(1.6 \times 10^{-19})(1350 \times 10^{17} + 480 \times 2.25 \times 10^3)}$$

$$\rho = \frac{1}{21.6} = 0.046\ \Omega cm$$

For R

$$R = \rho L / A$$

A :- $A = W \cdot t = 2 \times 10^{-6} \times 0.5 \times 10^{-6}$

$A = 1 \times 10^{-12}\ m^2$

$$R = 0.046 \times \frac{100 \times 10^{-6}}{1 \times 10^{-12}}$$

$$\boxed{R = 4.6 \times 10^{-18}\ \Omega}$$

Problem 4

For Boron doped mean Holes majority carriers

$\therefore P_0 = 10^{17}$ atoms/cm$^3$

$$n_0 = \frac{n_i^2}{P_0} = \frac{(1.5 \times 10^{10})^2}{10^{17}}$$

$$n_0 = 2.25 \times 10^3\ \text{cm}^{-3}$$

$$\boxed{n_0 = 2250\ \text{cm}^{-3}}$$

$$\rho = \frac{1}{q(\mu_n \cdot n_0 + \mu_p P_0)}$$

$$= \frac{1}{(1.6 \times 10^{-19})(1350 \times 2250 + 480 \times 10^{17})}$$

$$\boxed{\rho = 0.130\ \Omega\ \text{cm}}$$

$$R = \rho\, L/A$$

$$= 0.13 \times \frac{0.01}{(5 \times 10^{-5} \times 2 \times 10^{-4})}$$

$$\boxed{R = \frac{0.13 \times 1000000}{1.30 \times 10^5}\ \Omega}$$

Problem #5:-

for $10^{11}$ atoms/cm³.

$n_0 = 10^{11}$ cm⁻³

$n_i = 1.5 \times 10^{10}$ cm⁻³

$p_0 = \frac{n_i^2}{n_0}$

$p_0 = \frac{(1.5 \times 10^{10})^2}{10^{11}} = 2.25 \times 10^9$ cm⁻³

$\rho = \frac{1}{q(\mu_n n_0 + \mu_p p_0)}$

$\rho = \frac{1}{1.9 \times 10^{-19}(1350 \times 10^{11} + 480 \times 2.25 \times 10^9)}$

$\rho = 38676.93 \ \Omega\text{cm}$

$R = \rho L/A$

$= \frac{0.01}{1 \times 10^{-8}} (38676.93)$

$K = 3.86 \times 10^{10} \ \Omega$

Q6 Problem.

$\Delta = 10^{14}$ atom/cm²

$t = 1\mu m = 1 \times 10^{-4}$ cm

doping concentration

$C = \Delta/t$

$C = \frac{10^{14}}{1 \times 10^{-4}} = 10^{18}$ atom/cm³

## Problem 7

$R_\Delta = 50\Omega/\square$

$L = 10\mu m = 10\times10^{-6} m = 1\times10^{-3} cm$

$W = 0.5\mu m = 0.5\times10^{-6} m = 5.0\times10^{-5} cm$

$t = 0.3\mu m = 0.3\times10^{-6} m = 3\times10^{-5} m$

$n_i = 1.5\times10^{10} cm^{-3}$ (Silicon).

$$\rho = \frac{RA}{L}$$

$$R = \rho L/A$$

$$\rho = \frac{RA}{L} = \frac{50\times(3\times10^{-5})(5\times10^{-5})}{1\times10^{-3}}$$

$$\rho = 7.5\times10^{-5}\ \Omega cm$$

Since $\rho = \frac{1}{q(\mu_p p_0)}$

$\therefore p_0 = \frac{1}{q(\mu_p)\rho}$

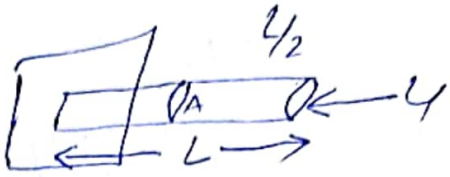
$$p_0 = \frac{1}{1.9\times10^{-19}\times 480\times 7.5\times10^{-5})}$$

$p_0 = 1.4619\times10^{20}\ cm^{-3}$

$n_i^2 = n_0 p_0$ $\therefore$ $n_0 = \frac{n_i^2}{p_0}$

$$n_0 = \frac{(1.5 \times 10^{10})^2}{1.46 \times 10^{20}}$$

$$\boxed{n_0 = 1.539} \text{ cm}^{-3}$$




## Problem 13

Reaction at anchored end

① Reaction force at the anchored end

$$R_{(anchored)} = -U$$

② Torque

$$T_{(anchored)} = 0$$

Forces and Torque at section A

① Axial Force at section A

$$F_A = U$$

② Torque at section A

$$T_A = 0$$

## Problem 14

At anchored End

① Reactive Force

$$R_{anchored} = -U_i - F_j$$

② $T_{(anch)} = F \cdot L$

At section

1. Axial Force

$$F_{axial} = U$$

2. Prep. Force $F_{prep} = F$.

3. Torque $T_A = F \times L/2$

## Problem 21

$L = 100\,\mu m = 0.01\,cm$

$W = 5\,\mu m = 5 \times 10^{-4}\,cm$

$t = 1\,\mu m = 1 \times 10^{-4}\,cm$

$n_0 = 10^{16}\,cm^{-3}$

$\mu_n = 1350\,cm^2/V.s$

$\mu_p = 480\,cm^2/V.s$

$n_i = 1.5 \times 10^{10}\,cm^{-3}$

$q = 1.9 \times 10^{-19}\,C$

Find (i) $I = ?$ (ii) Resistance $R = ?$

Solution $I = \frac{Wt^3}{12} = (5 \times 10^{-4})(1 \times 10^{-4})$.

$$\boxed{I = 4.16 \times 10^{-21}\,cm^4}$$

$R = \rho L/A$

So $\rho = \frac{1}{q \cdot n_0 \mu_n} = \frac{1}{(1.6 \times 10^{-19}) \times (1350 \times 10^{16})}$

$\rho = 0.46\,\Omega.cm$

$R = \rho \frac{L}{A} = 0.462 \times \frac{0.01}{(5 \times 10^{-4})(1 \times 10^{-4})}$

$R_2 = 0.2400\ \Omega$

## Problem 24

$b = 20\mu m = 20 \times 10^{-6} m$

$k = = 10\ N/m$

$f = 10\ kHz = 10^4\ Hz$

$E = 120\ GPa = 120 \times 10^9\ Pa$

Density of silicon

$\rho = 2330\ kg/m^3$

$g = 9.8\ m/s^2$

$$f_n = \frac{22.4}{2\pi}\sqrt{\frac{ET^2}{12\rho L^4}}$$

$$L^4 = \frac{22.4^2}{(2\pi f_n)^2} \cdot \frac{ET^2}{12\rho}$$